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CHAPTER

# Vector Algebra

**AIEEE Syllabus :** Vectors and Scalars, addition of vectors, components of a vector in two dimensional and three dimensional space, scalar and vector products, scalar and vector triple product. Application of vectors to plane geometry.

Vector algebra deals with addition / subtraction / product of vector quantities. Application of vectors to many geometrical problems cuts short the procedure. Hence geometrical significance of vectors should be well understood.

## VECTORS AND SCALARS

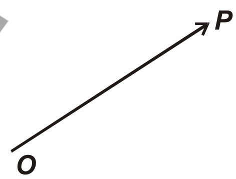
The physical quantities (we deal with) are generally of two types:

**Scalar Quantity:** A quantity which has magnitude but no sense of direction is called Scalar quantity (or scalar), *e.g.,* mass, volume, density, speed etc.

**Vector Quantity:** A quantity which has magnitude as well as a sense of direction in space is called a vector quantity, *e.g.,* velocity, force, displacement etc.

## NOTATION AND REPRESENTATION OF VECTORS

Vectors are represented by $\vec{a}, \vec{b}, \vec{c}$ and their magnitude (modulus) is represented by *a, b, c*, or $|\vec{a}|, |\vec{b}|, |\vec{c}|$,..... The vectors are represented by directed line segments.




For example, line segment $\overrightarrow{OP}$ represents a vector with magnitude *OP* (length of line segment), arrow denotes its direction. *O* is initial point and *P* is terminal point.

## SOME SPECIAL VECTORS

1. **Null vectors:** A vector with zero magnitude and indeterminate direction, denoted by $\vec{O}$.

2. **Unit vector:** A vector with unit magnitude (one unit), denoted by $\hat{a}$ where $|\hat{a}| = 1$ unit.

3. **Equal vectors:** Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal if they have same sense of direction and $|\vec{a}| = |\vec{b}|$, denoted by $\vec{a} = \vec{b}$.

4. **Like and unlike vectors:** Vectors having same sense of directions are called Like vector and opposite sense of directions are called Unlike vector.

5. **Negative of a vector:** Negative of a vector $\vec{a}$, denoted by $-\vec{a}$, is a vector whose magnitude is $|\vec{a}|$ and direction is opposite of $\vec{a}$.

6. **Collinear vectors:** Vectors having same line of action.

7. **Parallel vectors:** Vectors having same line of action or are parallel to a fixed straight lines.

8. **Coplanar vectors:** The vectors which lie in the same plane. At least three coplanar unequal vectors are required to make the sum zero and at least four if non-coplanar.

9. **Free vectors:** A vector not restricted to pass through a fixed point.

10. **Localized vectors:** A vector restricted to pass through a fixed point.

11. **Co-initial vectors:** Vectors having same initial point.

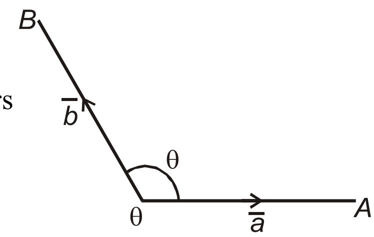
12. **Position vectors:** Let $O$ be fixed point in space, then vector $\overrightarrow{OP}$ ($P$ is any point in space) is called position vector of $P$ w.r.t. $O$. If $A$ and $B$ are any two point in space then

$$\overrightarrow{AB} = p.v. \text{ of } B - p.v. \text{ of } A = \overrightarrow{OB} - \overrightarrow{OA}.$$

## ANGLE BETWEEN TWO VECTORS

The angle between two vectors $\bar{a}$ and $\bar{b}$ represented by $OA$ and $OB$ is $\angle AOB = \theta$

$0 \le \theta \le \pi$

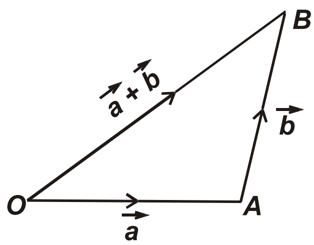



If $\theta = \frac{\pi}{2}$, then vectors are called orthogonal or perpendicular vectors

if $\theta = 0$ or $\pi$ then vectors are called parallel or coincident vectors.

## ADDITION OF VECTORS

Let $\vec{a}$ and $\vec{b}$ be two vectors. Draw $\overrightarrow{OA}$ representing vector $\vec{a}$. Taking terminal point of $\vec{a}$ as initial point of vector $\vec{b}$, draw $\overrightarrow{AB}$ representing vector $\vec{b}$. Then vector $\overrightarrow{OB}$ is called sum of vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a} + \vec{b}$ (triangle law of addition)




### Triangle Law of Addition

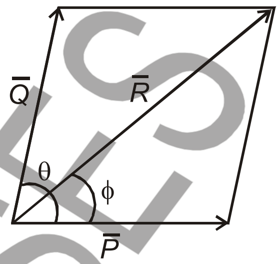
Three vectors are in equilibrium if represented by, three sides of a closed triangle taken in order, (in magnitude and direction).

**Converse of triangle law is also true.**

**Law of polygons** : If several vectors, when added, form a closed polygon, their resultant is zero.

**Parallelogram law of addition** – If two vectors are represented in magnitude and direction, by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction, by the co-initial diagonal of that parallelogram.

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ and $\tan\phi = \dfrac{Q\sin\theta}{P + Q\cos\theta}$




If $\theta = 0^\circ$ then $R = P + Q$ (maximum)

$\theta = \pi$ then $R = P \sim Q$ (minimum)

If $\theta = \dfrac{\pi}{2}$ then $R = \sqrt{P^2 + Q^2}$ and $\tan\phi = \dfrac{Q}{P}$

### Properties of Vector Addition

1. Vector addition is commutative.

   $\overline{a} + \overline{b} = \overline{b} + \overline{a}$

2. Vector addition is associative

   $(\overline{a} + \overline{b}) + \overline{c} = \overline{a} + (\overline{b} + \overline{c})$

3. $\overline{O}$ or null vector is additive identity.

   $\overline{A} + \overline{O} = \overline{A} = \overline{O} + \overline{A}$

4. Additive inverse of $\overline{A}$ is $(-\overline{A})$, Since

   $\overline{A} + (-\overline{A}) = \overline{O}$

## SUBTRACTION OF TWO VECTORS

The subtraction of two vectors $\overline{a}$ and $\overline{b}$ is defined as $\overline{a} - \overline{b} = \overline{a} + (-\overline{b})$

i.e. the vector to be subtracted is reversed and added to other vector.

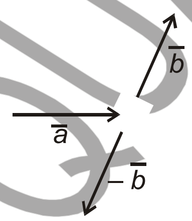
$\overline{a} - \overline{b} \neq \overline{b} - \overline{a}$ not commutative

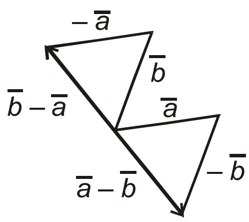
$\overline{a} - \overline{b} = -(\overline{b} - \overline{a})$ *i.e.* directions are opposite but magnitudes are same

For non-zero vectors $\overline{a}$ & $\overline{b}$

$\overline{a} + \overline{b} \neq \overline{a} - \overline{b}$

$|\overline{a} + \overline{b}| = |\overline{a} - \overline{b}|$ mean $\overline{a} \perp \overline{b}$

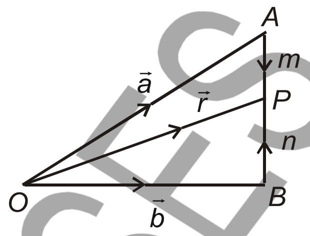


## SECTION FORMULAE

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$ then p.v. of point $P$ which divides $AB$ internally in the ratio $m : n$ given by

$$\overrightarrow{OP} = \vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n} \quad (m \neq -n)$$




If $P$ divides $AB$ externally in the ratio $m : n$ then

$$\overrightarrow{OP} = \frac{n\vec{a} - m\vec{b}}{n-m}$$

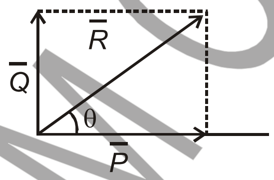
## VECTOR RESOLUTION OF A VECTOR (COMPONENTS OF A VECTOR)

A vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ has its $x$, $y$ and $z$ components as $a$, $b$ & $c$ respectively.

The vector resolution can be orthogonal or non-orthogonal. Orthogonal resolution means the components of the vector are mutually perpendicular otherwise non-orthogonal. Orthogonal components of vector $\overline{R}$ are as shown.

$P = R\cos\theta$ $\qquad \vec{R} = \overline{P} + \overline{Q}$

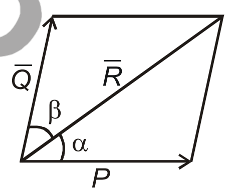
$Q = R\sin\theta$




Non-orthogonal components of $\overline{R}$ are as shown

$$P = \frac{R\sin\beta}{\sin(\alpha+\beta)}$$

$$Q = \frac{R\sin\alpha}{\sin(\alpha+\beta)}$$




## LINEAR COMBINATION OF VECTORS

If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then any vector $\vec{r}$ in the plane of $\vec{a}$ and $\vec{b}$ is uniquely expressed as $\vec{r} = x\vec{a} + y\vec{b}$, where $x$ and y are scalars. (Similar linear combination (unique) exists for three non-coplanar vectors, $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$ ).

## LINEARLY DEPENDENT AND INDEPENDENT VECTORS

Vectors $\vec{a}_1 \vec{a}_2, ...., \vec{a}_n$ are said to be linearly dependent if for scalars $x_1, x_2, ....x_n$ (not all zero), $x_1\vec{a}_1 + x_2\vec{a}_2 + .... + x_n\vec{a}_n = \vec{0}$ . If $x_1\vec{a}_1 + .... + x_n\vec{a}_n = \vec{0} \Rightarrow x_1 = x_2 = .... x_n = 0$ then vectors are called linearly independent.

***Note :*** *1. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then the vectors are linearly independent.*

2. *If* $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ *and* $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ *are linearly dependent then*

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

3. *If* $\vec{a}$, $\vec{b}$, $\vec{c}$ *are non-coplanars, then* $x_1\bar{a} + y_1\bar{b} + z_1\bar{c}$, $x_2\bar{a} + y_2\bar{b} + z_2\bar{c}$ *and* $x_3\bar{a} + y_3\bar{b} + z_3\bar{c}$ *are coplanar iff.*

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0$$

4. *Two non-zero, non-collinear vectors, or any three non-coplanar vectors are linearly independent.*
5. *Two collinear or any three coplanar or any four vectors in 3-D are linearly dependent.*
6. *Three points with position vectors* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *are collinear iff there exist scalars x, y and z (not all zero) such that* $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$ *where x + y + z = 0.*
7. *Four points with position vectors* $\vec{a}, \vec{b}, \vec{c}$ *and* $\vec{d}$ *are said to be coplanar iff there exist scalars x, y, z and w (not all zero) such that* $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = \vec{0}$ *where x + y + z + w = 0*

## Multiplication of a Vector by a Scalar

If *m* is a scalar and $\vec{a}$ is a vector, then $m\vec{a}$ (scalar multiple) is a vector whose magnitude is $|m||\vec{a}|$ and direction is same as of $\vec{a}$ (if *m* +ve) and opposite that of $\vec{a}$ if *m* is –ve.

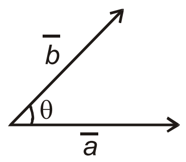
**Properties**

1. $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$
2. $(mn)\vec{a} = m(n\vec{a}) = n(m\vec{a})$
3. $(m + n)\vec{a} = m\vec{a} + n\vec{a}$

## Scalar (dot) Product of Two Vectors

The scalar product of vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}.\vec{b}$, is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, θ is angle between two vectors.

**Properties :**




1. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$
2. $\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$
3. $(m\vec{a}).\vec{b} = m(\vec{a}.\vec{b}) = \vec{a}.(m\vec{b})$
4. If $\theta = 0 \Rightarrow \vec{a}.\vec{b} = |\vec{a}||\vec{b}|$ (like vectors)
5. If $\theta = \pi \Rightarrow \vec{a}.\vec{b} = -|\vec{a}||\vec{b}|$ (unlike vectors)
6. If $\hat{a}$ and $\hat{b}$ are unit vectors then $\hat{a}.\hat{b} = \cos\theta$ (where θ is angle between them).
7. $\vec{a}.\vec{a} = |\vec{a}|^2 \Rightarrow |\vec{a}| = \sqrt{\vec{a}.\vec{a}}$
8. If $\vec{a} \perp \vec{b} \Rightarrow \vec{a}.\vec{b} = 0$ but $\vec{a}.\vec{b} = 0 \Rightarrow \vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a} \perp \vec{b}$.
9. If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the rectangular coordinate are *OX*, *OY* and *OZ* then $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$, $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$.
10. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

    $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

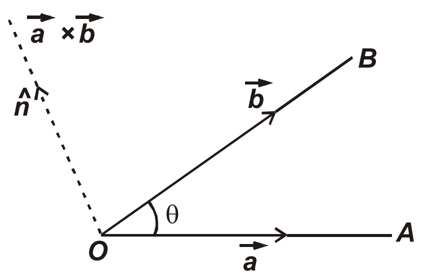
    and $\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$
11. Components of a vector $\vec{r}$ in the direction of vector $\vec{a}$ is $\left(\dfrac{\vec{r}.\vec{a}}{|\vec{a}|^2}\right)\vec{a}$ and $\perp$ to $\vec{a}$ is $\vec{r} - \left(\dfrac{\vec{r}.\vec{a}}{|\vec{a}|^2}\right)\vec{a}$.
12. Work done by a force $\vec{F}$ in displacing a particle from *A* to *B* $(\vec{AB} = \vec{d})$

    $W = \vec{F}.\vec{AB} = \vec{F}.\vec{d}$

## VECTOR (CROSS) PRODUCT OF TWO VECTORS

Vector product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ where θ is the angle between $\vec{a}$ and $\vec{b}$ whose direction is that of unit vector $\hat{n}$ which is $\perp$ to both $\vec{a}$ and $\vec{b}$ in such away that $\vec{a}, \vec{b}, \hat{n}$ form a right handed triad (right handed screw system).

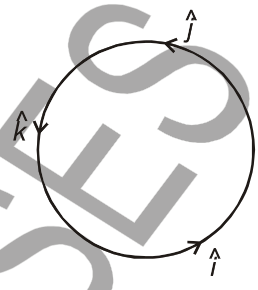
**Properties**




1. In general, $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$. In fact $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.
2. For scalar *m*, $m\vec{a} \times \vec{b} = m(\vec{a} \times \vec{b}) = \vec{a} \times m\vec{b}$.
3. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

4. If $\vec{a} \parallel \vec{b}$ then $\theta = 0$ or $\pi$ $\Rightarrow$ $\vec{a} \times \vec{b} = \vec{0}$ (but $\vec{a} \times \vec{b} = \vec{0} \Rightarrow \vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ $\vec{a} \parallel \vec{b}$ ). In particular $\vec{a} \times \vec{a} = 0$.

5. If $\vec{a} \perp \vec{b}$ then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\hat{n}$ (or $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$ )




6. $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ and $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i}$ and

   $\hat{k} \times \hat{i} = \hat{j}$ (use cyclic system)

7. Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is given by $\pm \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

8. If $\theta$ is angle between $\vec{a}$ and $\vec{b}$ then $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

9. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

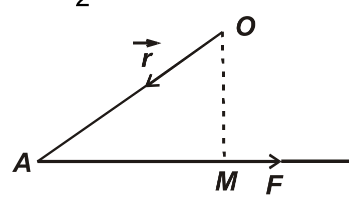
10. (i) If $\vec{a}$ and $\vec{b}$ are adjacent sides of a parallelogram, then

    Area of parallelogram = $|\vec{a} \times \vec{b}|$

    (ii) If diagonals of parallelogram are $\vec{a}$ and $\vec{b}$, then area of parallelogram = $\dfrac{1}{2}|\vec{a} \times \vec{b}|$

11. Vector moment of a force about a point:

    The vector moment or torque $\vec{M}$ of a force $\vec{F}$ acting at $A$ about the point $O$ is given by $\vec{M} = \vec{r} \times \vec{F}$




## SCALAR TRIPLE PRODUCT

**Scalar triple product:** If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors then $\vec{a}.(\vec{b} \times \vec{c})$ is called scalar triple product of $\vec{a}, \vec{b}$ and $\vec{c}$ denoted $[\vec{a}\ \vec{b}\ \vec{c}]$.

**Properties**

1. If $\vec{a}, \vec{b}$ and $\vec{c}$ are adjacent sides of a parallelopiped then volume = $[\vec{a}\ \vec{b}\ \vec{c}]$

2. $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$ (cyclic permutations of $\vec{a}, \vec{b}$ and $\vec{c}$ makes no change in value) but $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{a}\ \vec{c}\ \vec{b}]$ etc.

*i.e.*, Dot and cross can be interchanged, keeping the same cyclic order i.e. $\vec{a}.(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b}).\vec{c}$

3. For scalar *m*, $[m\vec{a}\,\vec{b}\,\vec{c}]=[\vec{a}\,m\vec{b}\,\vec{c}]=[\vec{a}\,\vec{b}\,m\vec{c}]=m[\vec{a}\,\vec{b}\,\vec{c}]$

4. The value of scalar triple product is zero if any two vectors are equal or parallel

5. If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then $[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

6. Condition of coplanarity: Three non-colinear, non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ are called coplanar iff $[\vec{a}\,\vec{b}\,\vec{c}]=0$

## Vector Triple Product

**Vector triple product:** If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors their vector triple product is defined by $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$ and $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}$

***Note*** : $\vec{a}\times(\vec{b}\times\vec{c})\neq(\vec{a}\times\vec{b})\times\vec{c}$

## APPLICATION OF VECTORS TO GEOMETRY

### 1. Vector Equation of Straight Line

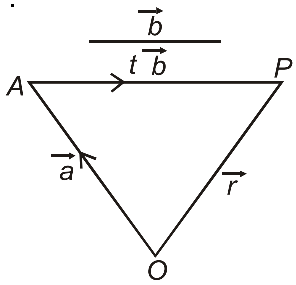
**Case I :** Vector equation of a straight line passing through a point $\vec{a}$ and parallel to vector $\vec{b}$. Let *O* be the origin and $\overrightarrow{OA}=\vec{a}$. Let *P* be any point on the line with $\overrightarrow{OP}=\vec{r}$.

Since $\overrightarrow{AP}\,||\,\vec{b}$, for some scalar *t,*

$$\overrightarrow{AP}=t\vec{b}$$

$\Rightarrow$ $\vec{r}-\vec{a}=t\vec{b}$

$\Rightarrow$ $\boxed{\vec{r}=\vec{a}+t\vec{b}}$ (required equation)




**Case II :** Vector equation of a straight line passing through two points $\vec{a}$ and $\vec{b}$.

$$\boxed{\vec{r}=\vec{a}+t(\vec{b}-\vec{a})}$$

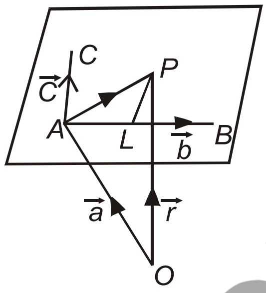
### 2. Vector Equation of a Plane

**Case I :** Vector equation of a plane passing through the point $\vec{a}$ and parallel to two given vectors $\vec{b}$ and $\vec{c}$. Let *O* be the origin, $\overrightarrow{OA}=\vec{a}$. Let *AB* and *AC* be the lines on the plane || to vectors $\vec{b}$ and $\vec{c}$. Let $P(\vec{r})$ be any point on the plane, draw *PL* || *AC,* (*AL* || *AB*).

Now $\overrightarrow{AL}=s\vec{b}$ and $\overrightarrow{LP}=t\vec{c}$ (*s, t* are scalars)

Now $\overrightarrow{AP} = \overrightarrow{AL} + \overrightarrow{LP} = s\vec{b} + t\vec{c}$

and $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \vec{a} + s\vec{b} + t\vec{c}$




Required equation $\boxed{\vec{r} = \vec{a} + s\vec{b} + t\vec{c}}$

Another form : $\boxed{[\vec{r}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]}$

**Case II :** Vector equation of a plane passing through the point $\vec{a}, \vec{b}, \vec{c}$ is given by

$\boxed{\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}}$ (where *s, t* are scalars)

Another form : $\boxed{\vec{r}.(\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}) = [\vec{a}\ \vec{b}\ \vec{c}]}$

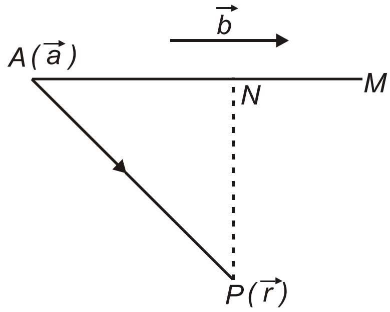
**Case III:** Vector equation of plane passing through a point $\vec{a}$ and perpendicular to vector $\vec{n}$ :

$$\boxed{(\vec{r} - \vec{a}).\vec{n} = 0}$$

**3. Perpendicular distance of a point from a line**

Let straight line passes through $A(\vec{a})$ and is || to $\vec{b}$. Then perpendicular distance of point $(\vec{r})$ from the

line = $\boxed{\dfrac{|(\vec{r} - \vec{a})\times\vec{b}|}{|\vec{b}|}}$




Another form : $\boxed{\left[(\vec{r} - \vec{a})^2 - \left\{\dfrac{(\vec{r} - \vec{a}).\vec{b}}{|\vec{b}|}\right\}^2\right]^{1/2}}$

**4. Perpendicular distance of a point from a plane**

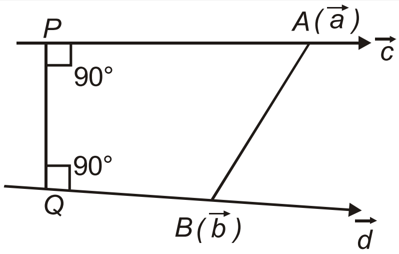
**Case I :** When plane passes through a point $\vec{a}$ and is || to $\vec{b}$ and $\vec{c}$. Distance of point $P(\vec{r})$ from the plane $\boxed{\dfrac{(\vec{r} - \vec{a}).(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}}$

**Case II :** When plane passes through the points $\vec{a}, \vec{b}$ and $\vec{c}$.

$$\boxed{\dfrac{(\vec{r} - \vec{a}).(\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b})}{|\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}|}}$$

**5. Shortest distance between two non-intersecting lines :**

Let there be two non-intersecting lines passing through $\vec{a}$ and $\vec{b}$ and are || to $\vec{c}$ and $\vec{d}$. If *PQ* be the shortest distance between them, then




$PQ$ = projection of $\overrightarrow{AB}$ on the vector $\vec{n}$ (where $\vec{n} = \vec{c} \times \vec{d}$ )

Then $PQ = \dfrac{(\vec{b} - \vec{a}).(\vec{c} \times \vec{d})}{|\vec{c} \times \vec{d}|}$

**6. Vector equation of the bisector of the angle between two straight lines :**

Let the two lines $AB$ and $AC$ meet at $A$ whose p.v. is $\vec{a}$. Let $\vec{b}$ and $\vec{c}$ be vectors || to $AB$ and $AC$ respectively. Also let $P$ be a point on internal bisector of $\angle BAC$ where $\overrightarrow{OP} = \vec{r}$.

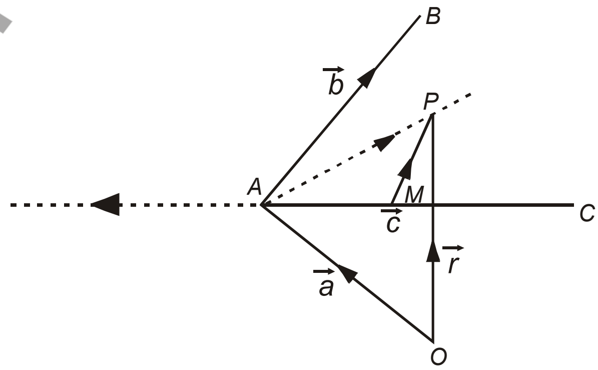
From $P$ draw line || $AB$ which meets $AC$ at $M$.

Now $\angle PAM = \angle PAB = \angle APM$

$\Rightarrow$ $AM = PM = t$ (say)

$\overrightarrow{AM}$ is collinear with $\vec{c}$ and $\overrightarrow{MP}$ is collinear with $\vec{b}$.

Then $\overrightarrow{AM} = t\dfrac{\vec{c}}{|\vec{c}|}, \overrightarrow{MP} = t\dfrac{\vec{b}}{|\vec{b}|}$




Now in $\Delta APM$, $\overrightarrow{AP} = \overrightarrow{AM} + \overrightarrow{MP} = t\left(\dfrac{\vec{c}}{|\vec{c}|} + \dfrac{\vec{b}}{|\vec{b}|}\right)$,

In $\Delta OAP$, $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$\Rightarrow$ $\vec{r} = \vec{a} + t\left(\dfrac{\vec{b}}{|\vec{b}|} + \dfrac{\vec{c}}{|\vec{c}|}\right)$, required equation of internal bisector

Equation of external bisector is given by $\vec{r} = \vec{a} + t\left(\dfrac{\vec{b}}{|\vec{b}|} - \dfrac{\vec{c}}{|\vec{c}|}\right)$